Electronic Power Control

VOLUME 2 :

ELECTRONIC MOTOR CONTROL

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To my wife Gilberte

PREFACE

This book first appeared in 1986 and after 29 years has reached the eighth edition. From the seventh edition the book was also available in English.

Every edition saw continuous updating rearranging as well as addition of material and chapters. At the same time attention was also paid to the didactic aspects. This is not just important for students but also for the large group of people who use the book for self study.

New in the eighth edition is a brief study of standing waves in transmission lines, of importance for a longue line between frequency converter and three phase motor.
Also new is an introduction to the principles of 3-level inverters.

In this edition we continue to use the tradition of white and green pages. The green pages contain the mathematical derivations which in the first case are not necessary for studying the electronics. Once a sufficiently high level and the desire for specialist knowledge the reader can choose to make use of the green pages without disturbing the continuity of the study.

To mention a few numerical details, this book contains more than seven hundred figures, a hundred photos and more than fifty fully worked problems.

The purpose of the book is to explain the principles and applications of power electronics. Electronic switches and converters are studied in volume 1 and drive technology and positioning systems are dealt with in volume 2.

The largest part of this book is distilled from more than 40 years of lessons, talks and projects. The most important source of information is my students, especially the few hundred of whom I was the mentor I guided during their thesis for Master of Applied Engineering Sciences.

These I quided in wich I remain thankful and indebted to them.

To my publisher Peter Laroy of Academia Press I wish to express my thanks for many years of pleasant cooperation.

Our thanks also goes out to Paul Fogarty from Rotterdam University for the accurate English translation.

I would also like to thank Prof. dr. ir. Bernard Baeyens of the Ibague University (Colombia) for correcting and improving the Spanish technical vocabulary.

Last but not least, we have to thank the advertisers. As a result of their support, we have been able tot minimize the recommended retail price (RRP) of our textbook.

In conclusion we wish the readers of this book a fruitful study.

Blankenberge, Belgium, September 2015

Jean.Pollefliet@telenet.be

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. **Semiconductor switches:** diodes, transistors, thyristors
. **Electronic power converters:** DC and AC controllers , choppers, SMPS, inverters
. **Applications of power electronics**
. **Computer simulations**

PRINCIPAL SYMBOLS

α	*transistor current gain*
α	*firing angle thyristor* (rad)
β	*conduction angle thyristor* (rad)
B	*magnetic flux density* (T = Wb/m²)
AC	*alternating current*
DC	*direct current*
δ	*duty ratio (%)*
e	*instantaneous e.m.f.* (V)
E	*RMS-value elektromotive force (e.m.f.)* (V) / *DC-e.m.f.* (V)
E	*electric field intensity* (V/m)
E_{on} ,E_{off}	*energy dissipation during transistor switching "on" and "off" respectively* (J)
f	*frequency* (Hz)
Φ	*flux per pole DC-machine / rotating air gap flux induction motor* (Wb)
Φ_{S1}	*flux one stator winding of an induction motor* (Wb)
g_{fs}	*transconductance* (Siemens / mho)
g_m	*transconductance coefficient* (Siemens / mho)
H	*magnetic field intensity* (A/m)
h_{FE}	*current gain common emitter connection*
i / $\hat{i}$	*instantaneous current* (A) / *peak value of sinusoidal current* (A)
i_0 / I_0	*output current of a circuit* (A)
i_μ	*magnetizing current* (A)
$\hat{i}_\mu$	*peak value of magnetizing current* (A)
I_{AV}	*average value of a semiconductor current* (A)
I_{RMS} / I	*r.m.s. value of current* (A) / *DC-current* (A)
J	*(polar) moment of inertia* (kgm²)
L_b	*load self inductance*
L_o	*magnetizing inductance (transformer / induction motor)* (H)
$\mathscr{L}$	*Laplace transform*
μ_0	*permeability of free space* ($4.\pi.10^{-7}$ H/m)
μ_r	*relative permeability*
M	*momentum (of torque)* (Nm)
M_{em}	*electromagnetic momentum(of torque)* (Nm)
M_J	*accelerating or decelerating momentum (of torque) due to inertia* (Nm)
$M_{max} = M_{po}$	*peak value momentum (of torque) induction motor* (Nm)
M_t	*total momentum of load torque (mechanical load M_L + static friction torque M_F + windage torques M_W ...)* (Nm)

R_μ	*reluctance* (A/Wb)
F_μ	*magnetomotive force (m.m.f.)* (Aw)
N_{Se}	*equivalent sinusoidal (stator) winding induction motor*
n	*motor speed* (r.p.m. or rad/s)
n_S	*synchronous speed (rotating stator field) induction motor* (r.p.m.)
n_R	*speed rotating rotor field induction motor* (r.p.m.)
η	*efficiency of operation (%)*
P	*DC-power (W) / average power* (W)
P_e	*eddy current loss density* (W/m³)
P_h	*hysteresis loss density* (W/m³)
p	*number of pole pairs DC-machine*
p	*number of pole pairs stator winding induction motor*
$\sigma_R.L_0$	*leakage inductance rotor induction motor*
$\sigma_S.L_0$	*leakage inductance stator induction motor*
R_b	*load resistance*
s	*Laplace operator*
T	*time period* (s)
T	*temperature* (°C ; K)
t_{on}	*time to switch on a power semiconductor (switch)* (µs; ns)
t_{off}	*time to switch off a power semiconductor (switch)* (µs; ns)
t_{ON}	*time that the power semiconductor is conducting (ON-state)* (µs ; ms)
t_{OFF}	*time that the power semiconductor is blocking (OFF-state)* (µs ; ms)
t_d	*delay time (to switch a transistor on)* (µs ; ns)
t_f	*fall time during switching off transistor* (µs ; ns)
t_r	*rise time during switching on transistor* (µs ; ns)
t_s	*storage time (to build off the space charge in a BJT)* (µs ; ns)
τ	*time constant* (s)
v	*instantaneous voltage* (V)
v_0 / V_0	*output voltage of a circuit* (V)
v_s / V_s	*supply voltage* (V)
$\hat{v}$	*peak value sinusoidal voltage* (V)
V	*voltage (DC, average, ...)* (V)
V_L / V_F	*line voltage / phase voltage in a three-phase system* (V)
V_{RMS}	*root mean square voltage* (V)
V_{di}	*(dc-) average voltage for ideal rectifier* (V)
$V_{di\alpha}$	*(dc-) average voltage for ideal controlled rectifier with firing angle α* (V)
v	*speed* (m/s)
W	*energy* (J)
ω	*angular frequency* (rad/s)

16 ELECTRIC MACHINES

16.1 TRANSFORMERS

CONTENTS

1.1 Transformer at no load

The simplest form of a single phase static transformer consists of a ferromagnetic circuit of Si-steel plates upon which two separate windings have been placed (fig. 16-1). The primary coil p has N_p windings and the secondary s has N_s windings. As long as no load is connected to the secondary we refer to the unloaded transformer or transformer at no-load. We now connect the voltage $v = \hat{v}_p \cdot \sin \omega t$ to the primary. A primary sinusoidal current flows, resulting in a sinusoidal flux in the core. The self inductance with respect to this flux Φ_0 is L_0. Primary current: $\approx \dfrac{V_p}{\sqrt{R_p^2 + \omega^2 \cdot L_0^2}}$

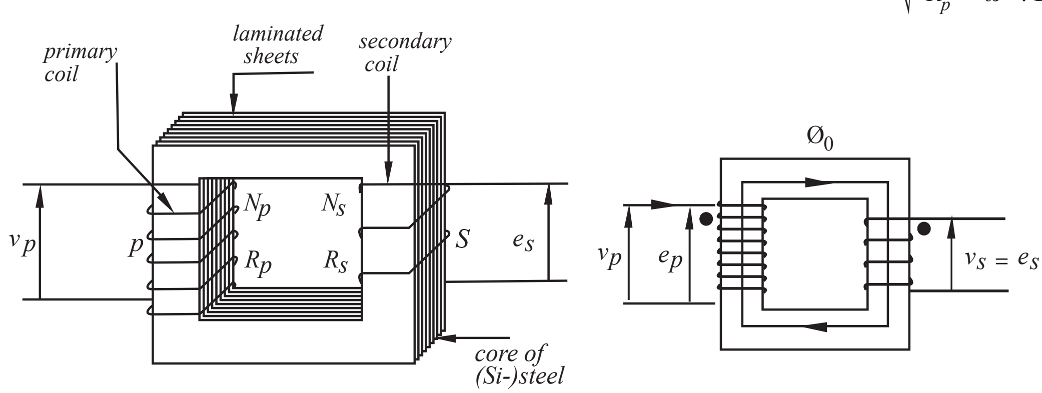

Fig. 16-1: Single phase transformer, no load

The primary current cannot yet be exactly determined since we first need to take the leakage reactance in the transformer into account. If we neglect the resistance R_p of the coil, then $I_\mu \approx \frac{V_p}{\omega . L_0}$ = magnetizing current. This wattless current lags 90° behind V_p (fig. 16-2) and creates the flux $\Phi_0 = \frac{N_p . I_\mu}{R_\mu}$. Here $R_\mu = \frac{l}{\mu_0 \, \mu_r \, A}$ is the reluctance of the magnetic circuit, with l being the average length of the field lines and A being the cross-sectional area of the core. I_μ is calculated later when we take R_p and the leakage flux into account.

Flux $\Phi_0 \rightarrow$

- is in phase with I_μ and $\pi/2$ behind $v_p = \hat{v}_p . \sin \omega.t \rightarrow \Phi_0 = \hat{\Phi}_0 . \sin(\omega t - \pi/2)$
- produces iron losses (see number 1.7.2): $P_{Fe} \approx V_p . I_v$; I_v is in phase with V_p
- $\vec{I}_\mu + \vec{I}_v = \vec{I}_n$ = no-load current, lags almost $\pi/2$ behind V_p
- induces an emf $e = N . \frac{d\Phi_0}{dt}$ in each coil

Primary:

- $e_p = N_p . \frac{\hat{\Phi}_0 . d \sin(\omega.t - \frac{\pi}{2})}{dt} = N_p . \omega . \hat{\Phi}_0 . \sin \omega.t$
- effective value: $$\boxed{E_p = \frac{N_p . \omega . \hat{\Phi}_0}{\sqrt{2}}} \quad (16\text{-}1)$$
- E_p is in phase with V_p and is the self induced emf of the primary.

Secondary:

- $e_s = N_s \frac{\hat{\Phi}_0 . d(\sin(\omega.t - \frac{\pi}{2}))}{dt} = N_s . \omega . \hat{\Phi}_0 . \sin \omega.t$
- effective value: $$\boxed{E_s = \frac{N_S . \omega . \hat{\Phi}_0}{\sqrt{2}}} \quad (16\text{-}2)$$

A secondary emf is produced with the same frequency as the applied primary voltage. The primary emf E_p is eliminated by the applied voltage V_p. If we ignore the voltage losses: $E_p = V_p$ = the applied primary voltage.

The magnetizing current can be written as: $$\boxed{I_\mu = \frac{E_p}{\omega.L_0}} \quad (16\text{-}3)$$

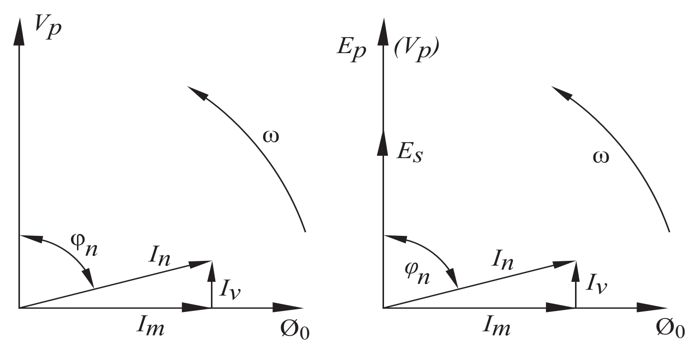

Fig. 16-2: Vector diagram of no-load transformer

From (16-1) and (16-2) follows: $\frac{E_p}{E_s}$ = transformer ratio $= \frac{N_p}{N_s}$

At no-load $E_p \approx V_p$ and $E_s = V_s$ so that $\frac{V_p}{V_s} = \frac{N_p}{N_s} = k =$ turns ratio (16-4)

In addition: $E_p = \frac{N_p \, . \, \omega \, . \, \hat{\Phi}_0}{\sqrt{2}} = N_p \, . \, \frac{2 \, . \, \pi \, . \, f}{\sqrt{2}} \, . \, \hat{\Phi}_0 = 4.44 \, . \, N_p \, . \, f \, . \, \hat{\Phi}_0$ (16-5)

By neglecting the losses, the flux is directly proportional to the primary voltage (assuming that the frequency is constant).

1.2 Transformer with load

1.2.1 Secondary and primary currents

When a load is connected to the secondary, then a current I_s flows. The power consumed by the secondary load is drawn from the net by the primary, which means that the current I_p is larger than the no-load current I_n .

With V_p constant, $E_p \approx V_p$ = constant. From (16-3) and (16-5), it follows that I_μ and Φ_0 are practically unchanged. Constant flux means unchanged iron losses (I_v) so that the current I_n also does not change. In other words, the secondary current I_s and the primary current I_p produce the same flux Φ_0 as I_n at no-load (see fig. 16-3a).

$$N_p \, . \, \vec{I}_p - N_s \, . \, \vec{I}_s = N_p \, . \, \vec{I}_n \rightarrow \vec{I}_p - \frac{\vec{I}_s}{k} = \vec{I}_n \rightarrow \vec{I}_p = \vec{I}_n + \frac{\vec{I}_s}{k} \quad (16\text{-}6)$$

From (16-6) the vectorial construction of $\vec{I}_p$ in fig. 16-3b follows.

Since with a good transformer I_n is quite small with respect to I_p , we find from (16-6) that

$$\vec{I}_p \approx \frac{\vec{I}_s}{k} \quad \text{or:} \quad \frac{I_p}{I_s} \approx \frac{1}{k} = \frac{N_s}{N_p} \quad (16\text{-}7)$$

From (16-7) and (16-4) it follows, by approximation: $\frac{V_p}{V_s} = \frac{I_s}{I_p} = \frac{N_p}{N_s} = k$ (16-8)

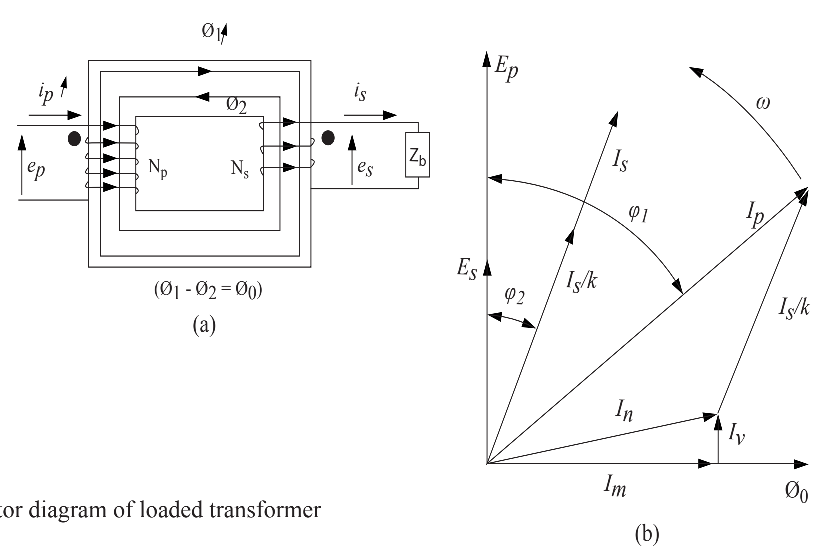

Fig. 16-3: Vector diagram of loaded transformer

1.2.2 Leakage flux

The current I_p produces a flux Φ_P which is mostly enclosed (Φ_1) within the core. A small part Φ_{pl} is not coupled with the secondary coil, so that $\Phi_P = \Phi_1 + \Phi_{pl}$. We refer to Φ_{pl} as the primary leakage flux. On the secondary side the current I_s produces a flux Φ_S which for the most part (Φ_2) flows through the primary and a small component Φ_{sl} (leakage flux) that is not linked to the primary: $\Phi_S = \Phi_2 + \Phi_{sl}$.

We therefore have: $\Phi_P = \dfrac{N_p \cdot I_p}{R_\mu}$ in phase with I_p and $\Phi_S = \dfrac{N_s \cdot I_s}{R_\mu}$ in phase with I_s .

In fig. 16-4, the instantaneous currents and voltages are drawn.

Here we see that Φ_1 and Φ_2 oppose each other, so that the resulting flux $\Phi_0 = \Phi_1 - \Phi_2$ = flux which was considered at no-load (= no-load flux !). This resulting flux Φ_0 is practically constant for every load and includes the emf's E_P and E_S as already shown.

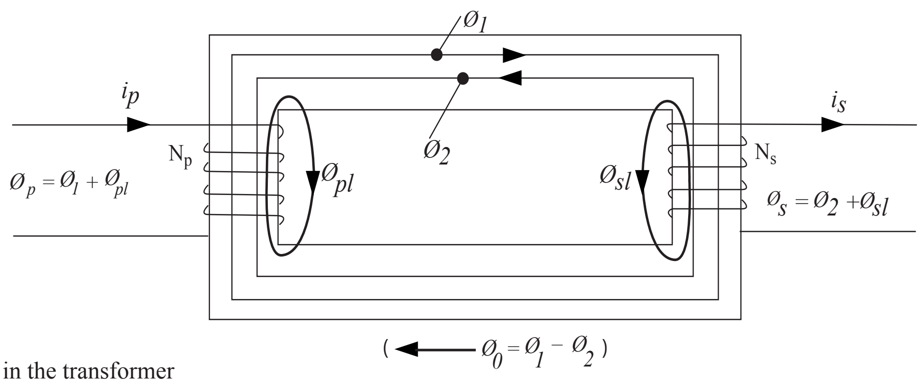

Fig. 16-4: Fluxes in the transformer

1.3 Transformer vector diagram

Primary:

- Leakage flux Φ_{pl} it's practically proportional to I_p : $\dfrac{N_p \cdot (\hat{\Phi}_{pl})}{\sqrt{2}} = s_p \cdot I_p$

  s_p = coefficient of primary self inductance with respect to leakage flux Φ_{pl}
- The leakage flux produces an emf in the primary coil: $e_{pl} = N_p \cdot \dfrac{d\,(\Phi_{pl})}{dt} = s_p \cdot \dfrac{d\,i_p}{dt}$
- If $i_p = \hat{i}_p \cdot \sin \omega.t$, then $e_{pl} = s_p \cdot \hat{i}_p \cdot \dfrac{d\,(\sin \omega.t)}{dt} = s_p \cdot \omega \cdot \hat{i}_p \cdot \sin(\omega.t + \frac{\pi}{2})$

  e_{pl}
  - is a sinusoidal emf , which leads I_P by 90°
  - effective value: $E_{pl} = \omega \cdot s_p \cdot I_p$

Summary:

Primary:

1. Induced counter-EMF E_P that leads Φ_0 by 90°
2. Induced counter-EMF $E_{pl} = \omega \cdot s_p \cdot I_P$ which leads I_P by 90°
3. Voltage drop $I_P \cdot R_P$ in phase with I_P
4. If we include the voltage losses $I_P \cdot R_P$ and $\omega \cdot s_P \cdot I_P$ we find:

$$\vec{V}_P = \vec{E}_P + \vec{E}_{pl} + \vec{I}_P \cdot R_P \qquad (16\text{-}9)$$

Secondary:
1. Flux Φ_0 produces an emf E_S which leads Φ_0 by 90°
2. Flux Φ_{sl} produces an emf $E_{sl} = \omega \cdot s_s \cdot I_S$ which leads I_S by 90°
3. Voltage drop $I_S \cdot R_S$ in phase with I_S
4. Terminal voltage V_S is formed by: $\vec{V}_S = \vec{E}_S - \omega \cdot s_s \cdot \vec{I}_S - \vec{I}_S \cdot R_S$ (16-10)

With what we have considered up to now, we can create a diagram in fig. 16-5 of a loaded transformer.

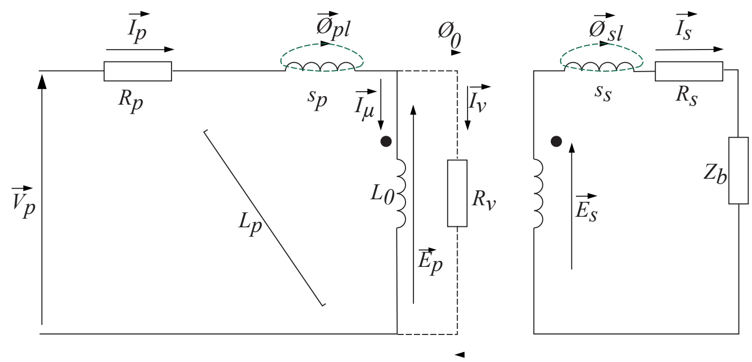

Fig. 16-5: Transformer with losses and secondary load

With the help of (16-9) and (16-10) we now construct fig. 16-6.

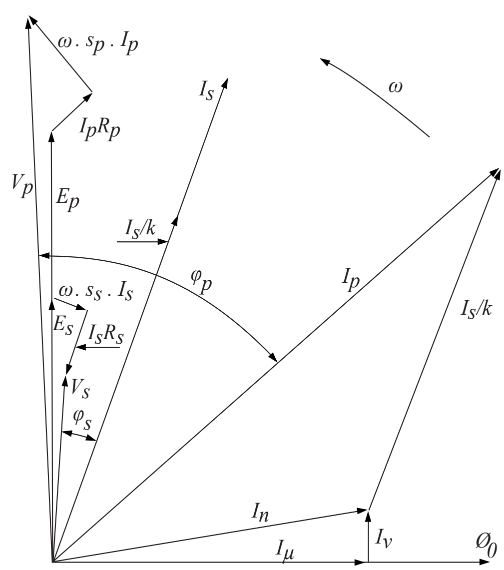

Fig. 16-6: Vector diagram of transformer with inductive load

Remark

From the figures 16-2 and 16-6 we see that the displacement angle between primary current and voltage decreases from almost 90° (at no-load) to a value determined by $\vec{I}_n$ and $\vec{I}_S$.

1.4 Impedance transformation

1.4.1 Transformation formula

Fig. 16-7a shows an ideal transformer, loaded with a series *R-L-C* circuit. An ideal transformer is a transformer without losses.

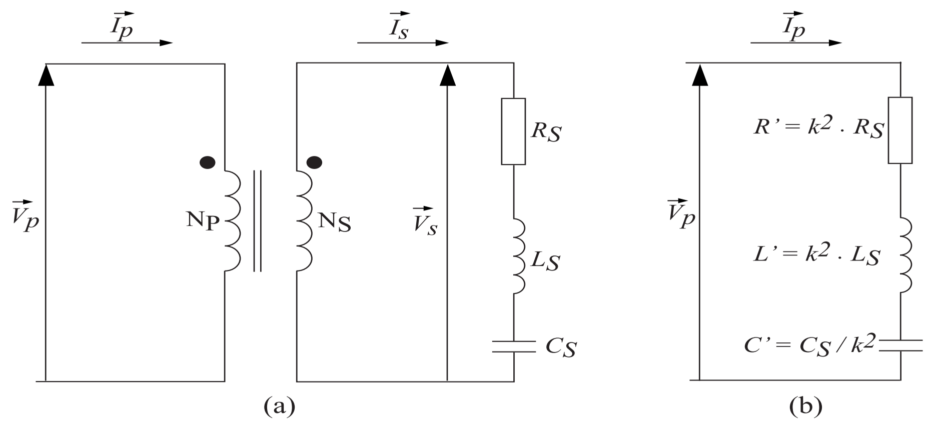

Fig. 16-7: Ideal transformer, loaded with a series *R-L-C* circuit: impedance transformation

Secondary: $v_s = R_s \cdot i_s + L_s \cdot \frac{di_s}{dt} + \frac{1}{C_s}\int_0^t i_p \cdot dt$

Application of (16-8) gives: $v_p = k^2 \cdot R_s \cdot i_p + k^2 \cdot L_s \cdot \frac{di_p}{dt} + k^2 \cdot \frac{1}{C_s}\int_0^t i_p \cdot dt$

For an ideal transformer, the secondary load can be represented as an equivalent circuit seen from the primary side (fig. 16-7b), as long as: $R' = k^2 \cdot R_s$; $L' = k^2 \cdot L_s$; $C' = C_S / k^2$.

More generally: an ideal transformer with secondary impedance $Z_{sec.}$ may be seen as a

primary impedance:

$$Z'_{prim.} = \left(\frac{N_p}{N_s}\right)^2 \cdot Z_{sec.} \quad (\Omega) \tag{16-11}$$

From (16-11) it follows that we can transform a primary impedance to an equivalent secondary

impedance:

$$Z'_{sec.} = \left(\frac{N_s}{N_p}\right)^2 \cdot Z_{prim.} \quad (\Omega) \tag{16-12}$$

1.4.2 Numeric example 16-1:

1. An electrical oven is supplied with 46 volt and has a power of 4 kW. The supply network is 230V-50 Hz. If we had an ideal transformer available, what is then:
   a) the transformation ratio
   b) the primary and secondary current
   c) impedance seen from the 230 V-50Hz net?

Solution:

a) $k = \frac{N_P}{N_S} = \frac{230}{46} = 5$

b) secondary current: $I_S = \frac{P_S}{V_S} = \frac{4000}{46} = 86.95\text{A}$

primary current: $I_P = \frac{I_S}{k} = \frac{86.95}{5} = 17.39\text{A}$

c) load impedance: $Z_S = \frac{V_S}{I_S} = \frac{46}{86.95} = 0.529 + \text{j}.0\ \Omega$

Transformed impedance seen from the source: $Z_{prim.} = k^2 \,.\, Z_{sec.} = 5^2$ x $0.529 = 13.23\Omega$

Proof: $Z_{prim.}\ x\ I_P = 13.23$ x $17.39 = 230\text{V}$!!

2. If maximum power transfer is required from the generator to the consumer, then the consumers impedance should be the complex conjugate value of the generator impedance.
We have a power amplifier with an output resistance of 48Ω and wish to connect a loudspeaker with the following characteristics: 30 W - 4Ω .
Maximum power transfer is possible by placing an impedance transformer between amplifier and loudspeaker. The turns ratio should be $k = \frac{N_P}{N_S} = \sqrt{\frac{Z_{prim.}}{Z_{sec.}}} = \sqrt{\frac{48}{4}} = 3.46$.

1.5 Magnetizing inductance

With a magnetising current I_μ the magnetic field strength in the core is $H = \frac{N_P \,.\, I_\mu}{l_k}$ and the magnetic induction is $B = \mu_0 \,.\, \mu_r \,.\, H$ so that the flux in the core of the transformer is:

$$\Phi_0 = B \,.\, A_k = \frac{\mu_0 \,.\, \mu_r \,.\, A_k}{l_k} \,.\, N_p \,.\, I_\mu$$

Whereby:
- Φ_0 (Wb): no-load flux ≈ resulting flux $(\Phi_1 - \Phi_2)$ with load
- B (Wb/m²): magnetic induction in the core
- μ_r : relative permeability of core material
- μ_0 : $= 4 \,.\, \pi \,.\, 10^{-7}$ H/m
- l_k (m): average length of field line in the core
- A_k (m²): cross-sectional area of core
- I_μ (A): magnetising current of the transformer.

If we call L_0 the self inductance of the primary with respect to the flux Φ_0 in the core, then we may write: $N_p \,.\, \Phi_0 = L_0 \,.\, I_\mu$ so that: $N_p \,.\, \Phi_0 = \frac{\mu_r \,.\, \mu_0 \,.\, A_k}{l_k} \,.\, N_p^2 \,.\, I_\mu = L_0 \,.\, I_\mu$

from which follows:

$$L_0 = N_p^2 \,.\, \frac{\mu_r \,.\, \mu_0 \,.\, A_k}{l_k} \quad \text{(H)} \qquad (16\text{-}13)$$

Numeric example 16-2:

1. A ring core transformer (fig. 16-8) consists of:

   Core: average radius 60 mm; cross-section of torus 45 mm; μ_r =1600.

   Insulation layer: 1 mm thick

   Primary: 3 layers: respectively 201, 189 and 140 windings AWG 18.

   Each layer is separated by 1 mm thick insulation.

   Insulation layer: 4 mm thick

   Secondary: two layers: 50 and 22 windings AWG 10, separated by 1 mm of insulation

2. Extract from winding wire table (AWG = American wire gauge)

AWG	diameter (with insulation) in mm min.	max.	resistance (per 100 m) Ω	admitted current (on base of 2A/mm²) A
10	2.64	2.69	0.3276	10.38
18	1.08	1.11	2.095	1.624

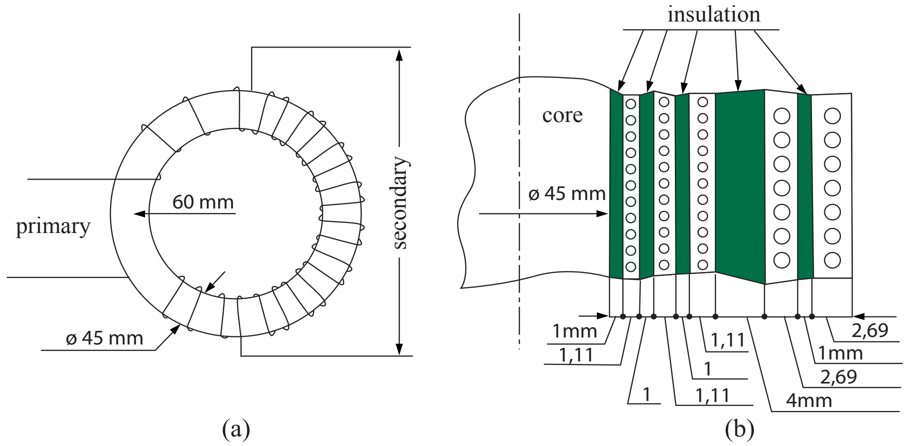

Fig. 16-8: Ring core transformer (a) cross-section (b) core and windings

Question:

1. Resistance of primary and secondary coil
2. Magnetising inductance

Solution:

1.1 Primary resistance

LAYER 1: length of one winding: π x 0.04811 = 0.15114 m

total length: 201 x 0.15114 = 30.38 m

LAYER 2: length of one winding: $\pi \cdot 0.05233 = 0.1644$ m
total length: $189 \cdot 0.1644 = 31.07$ m
LAYER 3: length of one winding: $\pi \cdot 0.05655 = 0.1776$ m
total length: $140 \cdot 0.1776 = 24.87$ m
Total length primary winding: 86.32 m

Resistance: $R_p = \frac{86.32}{100} \cdot 2.095 = 1.8\ \Omega$

1.2 Secondary resistance
LAYER 1: length of one winding: $\pi \cdot 0.06835 = 0.2147$ m
total length: $50 \cdot 0.2147 = 10.73$ m
LAYER 2: length of one winding: $\pi \cdot 0.07537 = 0.238$ m
total length: $22 \cdot 0.238 = 5.234$ m
Total length of secondary winding: 15.96 m

Resistance: $R_s = \frac{15.96}{100} \cdot 0.3276 = 0.0523\ \Omega$

2. Magnetizing inductance

$$L_0 = N_p^2 \cdot \frac{\mu_0 \cdot \mu_r \cdot A_k}{l_k} = 530^2 \cdot \frac{4 \cdot \pi \cdot 10^{-7} \cdot 1600 \cdot \pi \cdot 0.0225^2}{2 \cdot \pi \cdot 0.06} = 2.38\ \text{H}$$

1.6 Leakage inductance

From the viewpoint of voltage loss leakage inductance is undesirable. Transformers are therefore constructed to minimise the leakage fluxes. Fig. 16-9 shows, for example, how a coaxial implementation of primary and secondary coils minimises the leakage reactance by minimising the distance between consecutive coils. On the other hand, possible short circuit currents are limited by the leakage reactance, which can form a protection for the transformer. In practice distribution transformers are constructed with sufficient leakage reactance, so that short-circuit current is limited to 8 or 10 times the full load current.
In electronic power supplies ring core transformers are frequently used. Due to the construction method they have a minimum leakage reactance. Electronic technicians talk about “hard” transformers since large variations in the load coupled with low leakage inductance can produce large current spikes. These varying load conditions occur for example during commutation of one rectifier element to another on the secondary side of three-phase transformers.

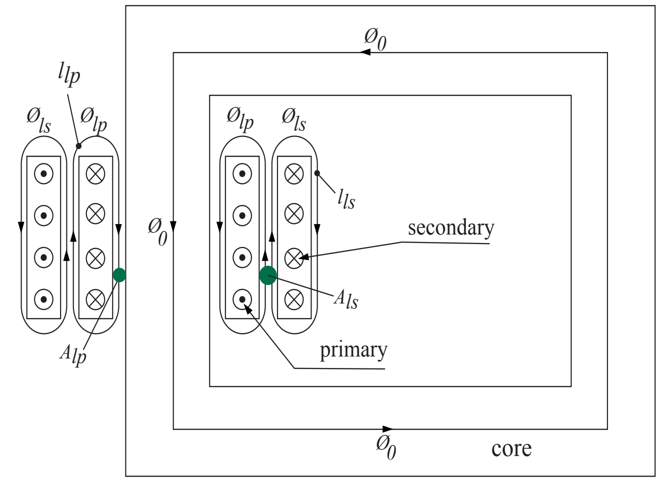

Fig. 16-9: Leakage fluxes by a coaxially wound transformer

To determine the leakage inductance, we consider the primary leakage flux (the same reasoning is valid for the secondary side). We can not make an accurate calculation since the cross-sectional area through which the flux flows can not be accurately determined. It is possible to make an approximate calculation. If the cross-sectional area where in the leakage flux flows is A_{lp} and the average length of the field line is l_{lp} then similar to expression (16-13), it may be written as:

$$L_{lp} = s_P = N_P^2 \cdot \mu_0 \cdot \frac{A_{lp}}{l_{lp}} \quad (16\text{-}14)$$

The field lines of the leakage flux complete their circuit through the air ($\mu_r = 1$) instead of through the ferromagnetic core (μ_r), which explains the difference with expression (16-13).

Numeric example 16-3:

We reuse the data of numeric example 16-2 ensure the possible parts of the leakage fluxes in fig. 16-10a and fig. 16-10b.

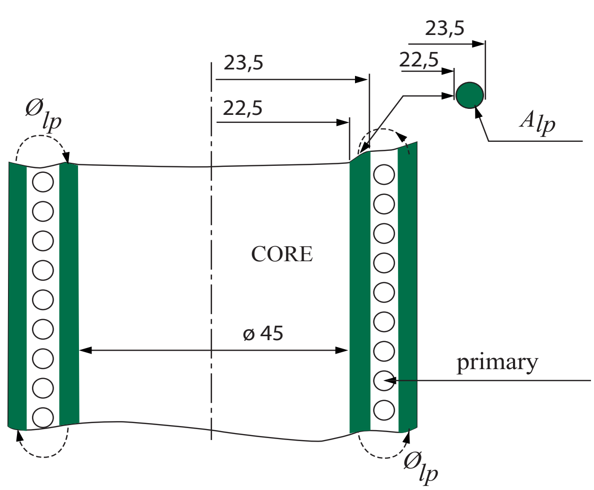

Fig. 16-10a: Primary leakage flux of transformer in fig. 16-8a

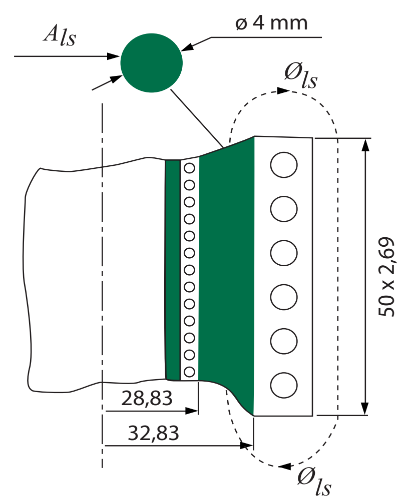

Fig. 16-10b: Secondary leakage flux fig. 16-8b

Primary leakage inductance

$$s_p = N_p^2 \cdot \frac{\mu_0 \cdot A_{lp}}{l_{lp}} = 530^2 \cdot \frac{4 \cdot \pi \cdot 10^{-7} \cdot \pi \cdot (23.5^2 - 22.5^2) \cdot 10^{-6}}{201 \cdot 1.11 \cdot 10^{-3}} = 228\ \mu H$$

Secondary leakage inductance

$$s_s = N_s^2 \cdot \frac{\mu_0 \cdot A_{ls}}{l_{ls}} = 72^2 \cdot \frac{4 \cdot \pi \cdot 10^{-7} \cdot \pi \cdot (32.83^2 - 28.83^2) \cdot 10^{-6}}{50 \cdot 2.69 \cdot 10^{-3}} = 37.42\ \mu H$$

If we realise that the magnetising inductance for this transformer is 2.38 H then we see that the leakage inductance is indeed minimal.

It is clear that the path of the leakage fluxes depends upon the practical implementation of the transformer windings. The present numeric example gives us a rough idea of the relative magnitude of the leakage inductance.

1.7 Energy losses

1.7.1. Copper losses

In the primary and secondary windings energy losses occur. If R_P and R_S are the respective resistances of the windings then the losses may be written as $R_P \cdot I_P^2$ and $R_S \cdot I_S^2$. The sum of both is the total energy loss. This is referred to as the copper losses of the transformer.

1.7.2 Iron losses

In ferromagnetic materials, subjected to a varying magnetic field, hysteresis losses occur:

$$P_h = k_h \cdot f \cdot \hat{B}^n \quad \text{W/kg} \qquad (16\text{-}15)$$

whereby: k_h = material constant of the ferromagnetic material used in relation to hysteresis losses.
f = frequency (Hz)
$\hat{B}$ = amplitude of the magnetic induction (T = Wb/m²)
n = empirical constant for the magnetic material ($1 < n < 3$).

Since the magnetic circuit of a transformer is constructed from metal plates, hysteresis losses occur. To limit these losses, it is desirable that the material constant be as small as possible.
A possibility in this case is an iron alloy using silicon (e.g. 3% silicon). If the core was made from solid iron, then considerable eddy currents would occur. These can be dramatically limited by making the magnetic circuit from plates which are insulated from each other and the surface of which is in the direction of the flux. As result of this the path of the eddy currents is limited.
The eddy current losses P_w can be determined with a formula in the following form:

$$P_w = k_w \cdot \delta^2 \cdot f^2 \cdot \hat{B}^2 \quad \text{W/kg} \qquad (16\text{-}16)$$

Here in : k_w = material constant with respect to the eddy current losses
δ = plate thickness in mm.

By adding silicon the electrical resistance is also increased as a result of which the eddy current losses are reduced. According to the last formula, it is advantageous to have the plates as thin as possible. Typical plate thickness lies between 0.3 and 1mm for 50 Hz operation. The plates can be 0.02 mm for high frequencies. For band wound cores thicknesses of 0.003 to 0.3mm are possible.

In many applications it is possible that non-sinusoidal waveforms occur. The eddy current losses are proportional to the square of the form factor $a = \frac{V_{RMS}}{V_{AV}}$ so that:

$$P_w = \frac{k_w}{1.11^2} \cdot a^2 \cdot \delta^2 \cdot f^2 \cdot \hat{B}^2 \qquad (16\text{-}17)$$

1.11 = form factor of sinusoidal voltage
a = form factor of the actual voltage.

Hysteresis and eddy current losses form the iron losses. They are sometimes called the constant losses of the transformer since they do not depend upon the load but only the magnetic induction. The magnetic induction only depends upon the applied voltage. The following table provides an idea of the iron losses with plates between 0.2 and 0.5 mm thick, and a frequency of 50 Hz with an induction of 1 Tesla.

Material	Losses in W/kg
commercial iron	5 ... 10
Si-Fe, warm rolled	1 ... 3
Si-Fe, cold rolled and crystal orientated	0.3 ... 0.6
50% Ni-Fe	0.2
approximately 65% Ni-Fe	0.06

Fig. 16-11 shows the total iron losses at 50 Hz for toroidal band wound cores of 0.3mm (data for cold rolled 3% Si-Fe cores). In fig. 16-12, we see the influence of the frequency on the total iron losses for the same material. Such cores are used for power transformers, impulse transformers, welding transformers, line transformers, etc... .

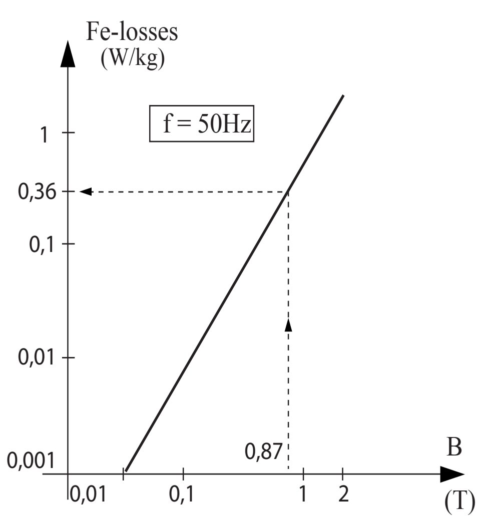

Fig. 16-11: Iron losses as a function of induction

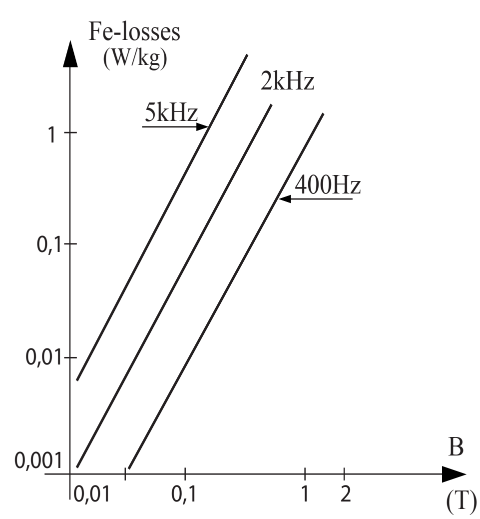

Fig. 16-12: Iron losses with the frequency as a parameter